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This report is an attempt to explain both s- and p-wave nonleptonic hyperon decays by means of the QCD enhanced effective weak Hamiltonian supplemented by the SU(3) Skyrme model used to estimate nonperturbative matrix elements. The model has only one free parameter, namely, the Skyrme charge e, which is fixed through the experimental values of the octet-decuplet mass splitting Δ and the axial coupling constant g_A. Such a dynamical approach produces nonleptonic hyperon decay amplitudes that agree with experimental data reasonably well.

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In the Skyrme model, baryons emerge as soliton configurations of pseudoscalar mesons [1]– [5]. Extension of the model to the strange sector, in order to account for a large strange quark mass, requires that appropriate chiral symmetry breaking terms should be included. The resulting effective Lagrangian can be treated by starting from a flavor symmetric formulation in which the existing kaon fields arise from rigid rotations of the classical pion field [3,6,7]. The associated collective coordinates are canonically quantized to generate states that possess the quantum numbers of physical strange baryons [3,6,5]. It turns out that the resulting collective Hamiltonian can be diagonalized exactly even in the presence of flavor symmetry breaking [4]. This approach leads to a good description of hyperon masses, charge radii, magnetic moments, etc. [5]. It should be noted that in the first phenomenological applications of the Skyrme model one attempted to fit absolute baryon masses, which required a ridiculously small pion decay constant [2,7]. Nowadays it is understood that there exist $1/N_c$ corrections to the total baryon masses that are not fully under control and therefore only mass *splittings* can be reliably reproduced. In this approach, f_π is kept at its experimental value. Hence the results for nonleptonic hyperon decays (NHD) [8] need to be updated accordingly.

This report is an attempt to test whether the effective weak Hamiltonian and the extended SU(3) Skyrme model are able to predict both s- and p-wave NHD amplitudes. This is done through straightforward calculations employing only the current low-energy elementary particle physics theory, i.e. the Standard Model. In this attempt we apply the Skyrme model-*minimal number of couplings*-concept to estimate the nonperturbative matrix elements of the 4-quark operators [8]. This approach uses only one free parameter, i.e., the Skyrme charge e. In order to avoid the unnecessary numerical burden, throughout this report we use the *arctan ansatz* for the Skyrme profile function [9].

Both s- and p-wave NHD amplitudes were quite successfully predicted by using quark models with QCD enhancement factors [10–12]. Note that there are not only current-algebra and ground-state exchange pole-diagram terms, but there exist other important contributions to both s and p waves. The so-called *factorizable* contributions and/or kaon poles were estimated in [10,11]. Pole-diagram contributions to p waves from the $(1/2^+)$-Rooper type of resonances and pole-diagram contributions to s-waves through the $(1/2^-)$-resonance exchange were calculated in [13].

The starting point of our analysis of NHD in the framework of the Standard Model [10] is the effective weak Hamiltonian in the form of the current $\otimes$ current interaction, enhanced by QCD. It is obtained by integrating out heavy-quark and W-boson fields. This Hamiltonian contains the 4-quark operators O_i and the well-known Wilson coefficients [10,11]. For the most recent values, see Ref. [12]. For the purpose of this paper, we use the Wilson coefficients from Ref. [11]: $c_1 = -1.90 - 0.61\zeta$, $c_2 = 0.14 + 0.020\zeta$, $c_3 = c_4/5$, $c_4 = 0.49 + 0.005\zeta$, with $\zeta = V^*_{td}V_{ts}/V^*_{ud}V_{us}$. Without QCD short-distance corrections, the Wilson coefficients would be $c_1 = -1$, $c_2 = 1/5$, $c_3 = 2/15$, and $c_4 = 2/3$. In this paper we simply consider both possibilities and compare the results.

The techniques used to describe NHD ($1/2^+ \to 1/2^+ + 0^-$ reactions) are known as a modified current-algebra (CA) approach. The general form is

$$< \pi(q)B'(p')|H_w^{eff}|B(p)> = \bar{u}(p')[\mathcal{A}(q) + \gamma_5\mathcal{B}(q)]u(p) = \frac{-i}{2f_\pi} < B'(p')|\hat{H}_w|B(p)> |_{q=0} + \mathcal{P}(q) + \mathcal{S}(q)\,. \tag{1}$$

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Here the first term is the CA contribution, the second is the modified pole term, and the third is a term that vanishes in the soft-meson limit. The $\mathcal{P}(q)$ term contains the contribution from the surface term, the soft-meson Born-term contraction, and the baryon pole term, which are combined in a well-known way [10,11]. It represents a continuation of the CA result from the soft-meson limit. Further continuation is contained in the factorizable term $\mathcal{S}(q)$, which is proportional to the meson four-momenta.

The parity-violating amplitudes $\mathcal{A}$ receive contributions A^c from CA commutator terms, factorizable terms $\mathcal{S}(q)$, and pole terms from the $(1/2^-)$ - resonance exchange. The main contributions to the $\mathcal{B}$ amplitudes come from the baryon pole terms $\mathcal{P}(q)$, including both the ground state and the radially excited states.

The current-algebra A^c and baryon-pole $B^{\mathcal{P}}$ amplitudes are well known from the literature. They contain weak matrix elements defined as $a_{BB'} =< B'|H_w^{PC}|B >$, which have the following general structure:

$$a_{BB'} = \sqrt{2} G_F V_{ud}^* V_{us} < B'|c_i O_i^{PC}|B > . \tag{2}$$

The factorizable term $\mathcal{S}(q)$ is calculated by inserting vacuum states. It is therefore a factorized product of two current matrix elements, where the first two-quark current is sandwiched between baryon states, while the second two-quark current is responsible for pion emission.

The CA and the baryon-pole terms contain the important 4-quark operator matrix elements, which are nonperturbative quantities. This is exactly the point at which the Skyrme model can be used. Each of the operators O_i from (2) contains four types of operators, namely, $\bar{d}u\bar{u}s\,, \bar{d}s\bar{u}u\,, \bar{d}s\bar{d}d,\, \bar{d}s\bar{s}s,$ and takes the form of the product of two Noether $SU(3)$ currents, which can be found in Refs. [8,14]. In our calculations we use four operators $\hat{O}_i$. The first of them is

$$\hat{O}_1 = \frac{1}{4} \bar{q}_L \gamma_\mu (\lambda_1 - i\lambda_2) q_L \, \bar{q}_L \gamma^\mu (\lambda_4 + i\lambda_5) q_L, \tag{3}$$

where the $SU(3)$ properties are expressed explicitly in terms of the Gell-Mann λ-matrices. The connection with the effective Hamiltonian operators O_i is obvious.

In order to estimate the matrix elements entering (2), we take the SU(3) extended Skyrme Lagrangian [5,14]:

$$\mathcal{L} = \mathcal{L}_\sigma + \mathcal{L}_{Sk} + \mathcal{L}_{SB} + \mathcal{L}_{WZ}, \tag{4}$$
$$\mathcal{L}_\sigma = \frac{f_\pi^2}{4} \int d^4x \, Tr \left(\partial_\mu U \partial^\mu U^\dagger \right), \quad \text{etc.}$$

where $\mathcal{L}_\sigma$, $\mathcal{L}_{Sk}$, $\mathcal{L}_{WZ}$, and $\mathcal{L}_{SB}$ denote the σ-model, Skyrme, symmetry breaking (SB), and Wess-Zumino (WZ) terms, respectively. For $U(x) \in SU(2)$, the SB and WZ terms vanish. The $f_\pi = 93$ MeV is the pion decay constant. Here the space-time-dependent matrix field $U(\vec{r}, t) \in SU(3)$ takes the form

$$U(\vec{r}, t) = A(t) \mathcal{U}(\vec{r}) A^\dagger(t), \tag{5}$$

where $\mathcal{U}(\vec{r})$ is the SU(3) matrix in which the Skyrme SU(2) *ansatz* is embedded:

$$\mathcal{U}(\vec{r}) = \begin{pmatrix} exp(i\vec{\tau}\cdot\vec{n}F(r)) & 0 \\ 0 & 1 \end{pmatrix}. \tag{6}$$

The time-dependent collective coordinate matrix $A(t) \in SU(3)$ defines the generalized velocities $A^\dagger(t)\dot{A}(t) = \frac{i}{2}\sum_{\alpha=1}^{8} \lambda_\alpha \dot{a}^\alpha$ and the profile function $F(r)$ is interpreted as a chiral angle that parametrizes the soliton.

In this work we use the *arctan ansatz* for $F(r)$ [9]:

$$F(r) = 2 \arctan \left[(r_0/r)^2 \right]. \tag{7}$$

Here r_0 - the soliton size - is the variational parameter and the second power of r_0/r is determined by the long-distance behavior of the massless equations of motion. After rescaling $x = ref_\pi$, one obtains $r_0/r = x_0/x$. The quantity x_0 has the meaning of a *dimensionless* size of a soliton and it is determined by minimizing the classical mass E_{cl}. Changing the variable $x = x_0 y$, all relevant integrals involving the profile function turn into an integral representation of the Euler beta functions, which can be evaluated *analytically*.

In the chiral limit of the $SU(2)$ Skyrme model, we obtain $x_0 = \sqrt{15}/4$ and the *arctan ansatz* reproduces static properties well [7]. For example, $g_A|_{e=4.1} = 1.25$.

In the $SU(3)$ extended Lagrangian (4) we have a new set of parameters, namely, $\hat{x} = 36.4$, $\beta' = -2.98 \times 10^{-5}\text{GeV}^2$, $\delta' = 4.16 \times 10^{-5}\text{GeV}^4$, determined from the masses and decay constants of the pseudoscalar mesons [5]. Owing to the presence of the β' and δ' terms in E_{cl}, x_0 becomes a function of e, f_π, β', and δ', and it is equal

$$x'^2_0 = \frac{15}{8}\left[1 + \frac{6\beta'}{f_\pi^2} + \sqrt{\left(1 + \frac{6\beta'}{f_\pi^2}\right)^2 + \frac{30\delta'}{e^2 f_\pi^4}}\,\right]^{-1}, \tag{8}$$

where we use the symbol x'_0 to distinguish it from the SU(2) case. After introducing the SB terms into the Lagrangian (4), one can either treat them as a perturbation [7] or one can try to sum up the perturbation series by numerically diagonalizing the resulting Hamiltonian [4]. In this paper we follow the perturbative approach. Indeed, as we show in the following, the SB effects on the decay amplitudes are very small.

At this point we can present the philosophy underlying the fitting procedure employed in this work. Since the couplings f_π and f_K are no longer free parameters fitted to the absolute values of the baryon masses, but are equal to their experimental values, the only remaining free parameter is the Skyrme charge e. The value $e \approx 4$ was successfully adjusted to the mass difference of the low-lying $1/2^+$ and $3/2^+$ baryons (Table 2.1 of [5]). This value of e was next employed to evaluate the static properties of baryons [5]. As explained above, in this work we use the SU(3) symmetric baryon wave functions in the spirit of the perturbative approach to SB.

For the evaluation of NHD, the important baryon static properties are the octet-decuplet mass splitting Δ and the axial coupling constant g_A. We compute these quantities by using the *arctan ansatz* in the $SU(3)$ extension of the Skyrme Lagrangian (4). However, if we fix Δ, the axial decay constant g_A is underestimated. This is a well-known problem of the Skyrme model, which can be cured in the more sophisticated chiral models involving quarks [15]. Therefore, we determine two values of the charge e through fixing Δ and g_A to their experimental values. The *arctan ansatz* gives

$$\Delta = \frac{3}{2\lambda_c(x'_0)}, \tag{9}$$

$$g_A = \frac{14\pi}{15e^2}\left(2x'^2_0 + \pi\right) + (1-\hat{x})\frac{16\pi\beta'}{225e^2 f_\pi^2}x'^2_0 + \frac{7\sqrt{2}N_c}{192 e f_\pi}\frac{x'_0}{\lambda_s(x'_0)}, \tag{10}$$

where

$$\lambda_c(x'_0) = \frac{\sqrt{2}\pi^2}{3e^3 f_\pi}\left(6(1+2\frac{\beta'}{f_\pi^2})x'^3_0 + \frac{25}{4}x'_0\right), \tag{11}$$

$$\lambda_s(x'_0) = \frac{\sqrt{2}\pi^2}{4e^3 f_\pi}\left(4(1+4\frac{\beta'}{f_\pi^2})x'^3_0 + \frac{9}{4}x'_0\right). \tag{12}$$

The quantity $\lambda_c(x'_0)$ represents the rotation moment of inertia in coordinate space, while the $\lambda_s(x'_0)$ is the moment of inertia for flavor rotations in the direction of the strange degrees of freedom. By fixing Δ and g_A to their experimental values, we obtain $e = 4.228$ and $e = 3.423$, respectively. In further calculations we use the mean value $e = 3.83$ and $x'_0|_{e=3.83} = 0.8789$, i.e., 10% less than in the massless case.

For the Lagrangian $\mathcal{L}$, we calculate the matrix element of the product of two $(V-A)$ currents between the octet states using of the Clebsch-Gordon decomposition [8]:

$$< B_2|\hat{O}^{(SK)}_{\bar{\alpha}\bar{\beta}}|B_1> \; = \Phi^{SK}\sum_{R,\Gamma,\bar{\varphi}} S^{SK}_{(\Gamma)}(R)\left(\begin{array}{cc|c} 8 & 8 & R \\ \bar{\alpha} & \bar{\beta} & \bar{\varphi}\end{array}\right)\left(\begin{array}{cc|c} R & 8 & 8_\Gamma \\ \bar{\varphi} & \bar{b}_1 & \bar{b}_2\end{array}\right). \tag{13}$$

The same holds for the WZ and SB currents. The total matrix element is then simply a sum $< \hat{O}^{(SK)}_i + \hat{O}^{(SB)}_i + \hat{O}^{(WZ)}_i >$, with $i = 1, \ldots, 4$. The quantities Φ are given by the overlap integrals of the profile function. Using the *arctan anzatz* (7), we obtain analytical expressions for the integrals as functions of x'_0:

$$\begin{aligned}
\Phi^{SK} &= 3\sqrt{2}\pi^2\left(2x'_0 + \frac{15}{2x'_0} + \frac{847}{64}\frac{1}{x'^3_0}\right)\frac{f_\pi^3}{e}, \\
\Phi^{WZ} &= \frac{231}{512}\frac{\sqrt{2}}{\pi^2}\frac{1}{x'^3_0}(e f_\pi)^3, \\
\Phi^{SB} &= (1-\hat{x})\,\beta'\frac{4\pi^2}{\sqrt{2}}\left(x'_0 + \frac{45}{8x'_0}\right)\frac{f_\pi}{e}.
\end{aligned} \tag{14}$$

For the $\hat{O}_1$ operator, $R = 8_{a,s}$ or 27. ; then

$$< p\uparrow|\hat{O}_1|\Sigma^+\uparrow> \;=-\frac{1}{4}\left(\frac{2}{25}|_8+\frac{1}{675}|_{27}\right)\Phi^{SK}-\frac{1}{4}\left(\frac{2}{25}|_8-\frac{1}{75}|_{27}\right)\Phi^{WZ}-\frac{1}{4}\left(\frac{7}{75}|_8+\frac{17}{1050}|_{27}\right)\Phi^{SB}. \tag{15}$$

The 27-piece is very small, which is an important proof of the octet dominance. For $f_\pi = 93$ MeV and $e = 3.83$, we obtain the following numerical values of the above integrals in units of GeV^3:

$$\Phi^{SK} = 0.262,\;\; \Phi^{WZ} = 0.004,\;\; \Phi^{SB} = 0.005. \tag{16}$$

From eqs.(15) and (16) we find the following structure for a typical matrix element:

$$< p\uparrow|\hat{O}_1|\Sigma^+\uparrow>= (-20.37\Phi^{SK} - 16.67\Phi^{WZ} - 27.38\Phi^{SB})10^{-3} = (-5.34|_{SK} - 0.06|_{WZ} - 0.14|_{SB})10^{-3}GeV^3. \tag{17}$$

It is clear that on top of the octet dominance we also find the dominance of the Skyrme Lagrangian currents over the WZ and SB currents in the evaluation of a typical weak matrix element between two hyperon states. For $e \approx 4$, the SB and WZ terms are of comparable size and their coherent contribution to (17) is below 4%.

The main aim of this report is to learn how the most common approach including the Skyrme model to estimate nonperturbative matrix elements applies to nonleptonic hyperon decays. Note that there exists a different approach of Ref. [16] in which meson-baryon couplings are directly obtained from the chiral Lagrangian. In Ref. [16] a new contact term is predicted for p-wave amplitudes. The correspondence between the two approaches may in principle be established providing a solution to the ambiguity of short-distance corrections.

In this work we have added factorizable, $A^{\mathcal{S}}(m_\pi^2)$ and $B^{\mathcal{S}}(m_\pi^2)$, contributions to the Skyrme model amplitudes $A^c(0)$ and $B^{\mathcal{P}}(m_\pi^2)$. The complete results are given in Table I. Comparison of the total amplitudes $\mathcal{A}(m_\pi^2)$ and $\mathcal{B}(m_\pi^2)$ with experiment shows the following:

(a) Short-distance corrections to the effective weak Hamiltonian are beyond doubt very important.

(b) Signs and order of magnitudes of all amplitudes are always correctly reproduced.

(c) s waves are in good agreement with experiment.

(d) The Pati-Woo theorem violation [17] and the 27-contaminations are found to be small. It is clear that the nonvanishing $\mathcal{A}(\Sigma^+_+)$ amplitude is still too small, in good accord with small values of the 27-contamination [19], and that additional contributions are needed [13].

(e) p waves are subject to some uncertainties. Namely, Donoghue et al. [16] showed that, in the Skyrme model, a contact term appeared and should be added to the results for p-waves. That has been taken care of in Ref. [14] and is not present in our case. In our opinion, $\mathcal{B}(m_\pi^2)$ amplitudes are not fully described by our formulas; nevertheless, they agree with experiment reasonably well.

(f) Finally, the factorizable contributions are small. Therefore, they represent just the fine tuning to the total $\mathcal{A}(m_\pi^2)$ and $\mathcal{B}(m_\pi^2)$ amplitudes.

Irrespectively of all these open questions and possible short-comings, it seems that the general dynamical scheme supported by the Skyrme model leads in a good direction. Obviously, we have not controlled all details; nevertheless, it seems that with the Skyrme model applied in the dynamical framework of this paper, or in the framework of a more complicated dynamics [14,16], we might be on the right track.

To conclude, we would like to emphasize the fact that the pure Skyrme model Lagrangian $\mathcal{L}$ cannot explain nonleptonic hyperon decays [14]. However, the QCD-corrected weak Hamiltonian H_w^{eff}, together with the inclusion of other possible types of contribution to the total amplitudes (K, K^*-poles, and /or factorization; $(1/2^{\pm *})$-poles, etc.) supplemented by the Skyrme model, leads to a correct answer. This would include the explanation of the octet dominance, the $|\mathbf{\Delta I}| = 1/2$ selection rule, $\mathcal{A}(\Sigma^+_+) \neq 0$, and the p/s-wave puzzle. Nevertheless, this is certainly a matter of another series of studies.

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[1] T.H.R. Skyrme, Proc. Roy. Soc. **A260**, 127 (1961).
[2] G. Adkins, C.R. Nappi and E. Witten, Nucl. Phys. **B228**, 552 (1983).
[3] E. Guadagnini, Nucl. Phys. **B236**, 35 (1984).
[4] H. Yabu and K. Ando, Nucl. Phys. **B301**, 601 (1988).
[5] H. Weigel, Int. J. Mod. Phys. **A11**, 2419 (1996); J. Schechter and H. Weigel, hep-ph/9907554 (1999).

[6] P.O. Mazur, M.A. Nowak and M. Praszałowicz, Phys. Lett. **147B**, 137 (1984); S. Jain and S. Wadia, Nucl. Phys. **B258**, 713 (1985).
[7] M. Praszałowicz, Phys. Lett. **158B**, 264 (1985); Acta. Phys. Pol. **B22**, 525 (1991).
[8] J. Trampetić, Phys. Lett. **144B**, 250 (1984); M. Praszałowicz and J. Trampetić, Phys. Lett. **161B**, 169 (1985); Fizika **18**, 391 (1986).
[9] D.I. Diakonov, V.Yu. Petrov and M. Praszałowicz, Nucl. Phys. **B323**, 53 (1989).
[10] D. Tadić and J. Trampetić, Phys. Rev. **D23**, 144 (1981); J.F. Donoghue, H. Golowich and B.R. Holstein, Phys. Rep. **131** , 319 (1986).
[11] J.F. Donoghue, H. Golowich and B.R. Holstein, *Dynamics of the Standard Model* (Cambridge Univ. Press, 1994).
[12] A.J. Buras, Les Houches Lectures; hep-ph/9806471.
[13] M. Milošević, D. Tadić and J. Trampetić, Nucl. Phys. **B207**, 461 (1982). S. Pakvasa and J. Trampetić, Phys. Lett. **126B**, 122 (1983).
[14] N.N. Scoccola, Phys. Lett. **B428**, 8 (1998); D. Dumm, A. Garcia and N. Scoccola, Phys. Rev. **D62**, 014001 (2000).
[15] M. Wakamatsu and T. Watabe, Phys.Lett.**B312**, 184 (1993); Ch.V. Christov et al, Phys.Lett. **B325**, 467 (1994); A. Blotz, M. Praszałowicz and K. Goeke, Phys.Lett. **B317**, 195 (1993).
[16] J.F. Donoghue, H. Golowich, and Y-C.R. Lin, Phys. Rev. **D32**, 1733 (1985); **D33**, 2728(E) (1986).
[17] J.C. Pati and C.W. Woo, Phys. Rev. **D3**, 2920 (1971).
[18] Rev. of Part. Prop., Eur. Phys. J. **C15**, 1 (2000).
[19] The $\mathcal{L}^{(27)}_{2+4}$ from Ref. [14] with the *arctan ansatz* as the profile function gives $A^{(27)}(\Sigma^+_+)|_{e=4.1} = 2.5 \times 10^{-9}$.

TABLE I. The s-wave ($\mathcal{A}$) and p-wave ($\mathcal{B}$) NHD amplitudes. Choices (off, on) correspond to the amplitudes without and with inclusion of short-distance corrections, respectively. For the sake of comparison, we have added the constituent quark-model evaluation of the A^c and $B^{\mathcal{P}}$ amplitudes [10].

$Amplitude(10^{-7})$		(Λ^0_-)	(Ξ^-_-)	(Σ^+_0)	(Σ^+_+)
$A^c(0)$	off	2.02	−2.94	−2.28	0.02
	on	3.84	−5.56	−4.34	0.04
$A^{\mathcal{S}}(m_\pi^2)$	off	0.03	−0.57	−0.49	0
[10]	on	−0.42	0.25	−0.01	0
$\mathcal{A}(m_\pi^2)$	off	2.05	−3.51	−2.77	0.02
(this work)	on	3.42	−5.31	−4.35	0.04
Exp. [18]		3.35	−4.85	−3.27	0.13
$A^c(0)$	off	0.78	−1.86	−1.36	0
CQM [10]	on	1.49	−3.53	−2.59	0
$B^{\mathcal{P}}_{(1/2^+)}(m_\pi^2)$	off	20.1	21.8	13.7	14.8
	on	38.1	41.4	25.9	28.2
$B^{\mathcal{S}}(m_\pi^2)$	off	3.6	−1.5	−0.4	0
[10]	on	6.0	−2.4	0.4	0
$\mathcal{B}(m_\pi^2)$	off	23.7	20.3	13.3	14.8
(this work)	on	43.4	38.2	26.3	28.2
Exp. [18]		22.3	17.4	26.6	42.2
$B^{\mathcal{P}}_{(1/2^+)}(m_\pi^2)$	off	2.9	7.8	7.3	10.4
CQM [10]	on	5.6	14.8	13.9	19.7